Substituting in (9), we have

$$w = \frac{c_0}{4\nu}\left[(a^2 - r^2) + (b^2 - a^2)\frac{\log(r/a)}{\log(b/a)}\right] + \sum_{n=1}^{\infty}\left[\left(\frac{a}{r}\right)^{1/2}\frac{\sin K(b-r)}{\sin K(b-a)} + \left(\frac{b}{r}\right)^{1/2}\frac{\sin K(r-a)}{\sin K(b-a)} - 1\right]\frac{c_n}{n}e^{-nt}$$

Reference

[1] Verma, P D, "On the flow of a viscous liquid under exponential pressure gradient superposed on the steady laminar motion of incompressible fluid between two co axial cylinders," Proc Natl Inst Sci India 26 (1–12), 266 (1960)

A Load-Sinkage Equation for Lunar Soils

DONALD L DEWHIRST*
Chrysler Co1po1ation, Det1oit, Mich

Nomenclature

p = pressure, load divided by footing area, psi
z = sinkage, in
k_c, k_ϕ = soil-sinkage moduli, dimensions depend upon n
n = dimensionless exponent related to shape of p1essure-sinkage curve
b = smallest dimension of footing area, e g , width of track or tire
a_1, a_2 = soil parameters, coefficients of polynominal

Introduction

SOIL models ranging from hard rock to fine dust[1–4] have been proposed for the lunar surface Since the fine-dust hypothesis presents the greater challenge to vehicle mobility, it is being studied somewhat extensively and is the only type of soil considered herein

It is convenient to fit load-sinkage data with a mathematical equation in order that calculations can be made regarding the resistance of the soil to vehicle motion and the amount of energy absorbed by the soil in its deformation This energy should be allowed for in fuel consumption estimates and in powerplant selection In addition, the form of the equation establishes parameters that may be used in a dimensional analysis and model study

Since the load-sinkage equation now in most common use,[5] $p = (k /b + k_\phi)z^n$, was selected as a best fit to a wide variety of soils, presumably including dry granular soil, silt, loam, clay, etc , it was considered desirable to compare this equation with the data available for presumed lunar-type soil, namely, the dry granular type

The data considered included five soils (tuff, scoria, rhyolite, pumice, and basalt) selected by Chrysler geologist Milton Schloss as being representative of possible lunar soils Their particle size ranged from 53 μ up to 3 36 mm They were tested in atmosphere, in moderate vacuum, and in very high vacuum In addition, data available in the literature were considered These included vacuum tests of pumice[6] and of white silica flour[7] Finally, data from atmospheric testing of dry sand and gravel were considered

The parameters in the equation $p = (k /b + k_\phi)z^n$ are normally obtained by plotting pressure-sinkage data from two different size footings on log-log paper and fitting a straight line to each set of data The slope of each line determines the value of the exponent n The lines are, of course, parallel if the equation fits the data The k and k_ϕ moduli are then found by solving simultaneous equations relating the p-z intercepts and the footing size A straightforward application of this procedure often leads to negative values of k for dry soils[8] Since the k /b term can be made large compared to k_ϕ for sufficiently small b, e g , a narrow tire, this term can introduce a large error Therefore, most investigators set $k = 0$ when considering this type of soil This is equivalent to stating that the pressure-sinkage relationship is independent of footing size The problem here is that the random variation of the soil properties is larger than the size effect, so that any attempt at extrapolation of the size effect is dangerous For these reasons, no size effect will be considered in this paper, and comparison is made between the equation in the form $p = k_\phi z^n$ and a polynomial

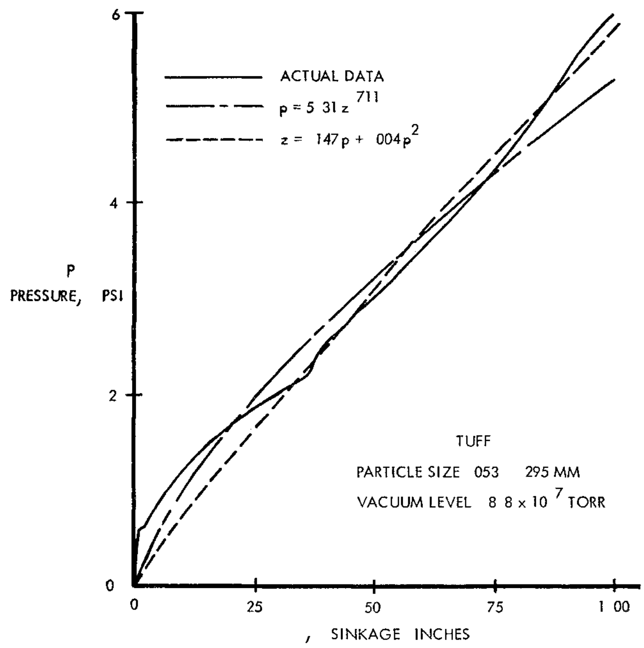

Fig. 1 Soil tests in moderate vacuum

By the Weierstrass approximation theorem,[9] it is known that any function continuous over some finite interval can be uniformly approximated by a polynomial of some degree to any desired accuracy The polynomial is then a useful approximation as long as the number of terms necessary to obtain the desired accuracy is not excessive In the following, a comparison is made between the power function $p = k_\phi z^n$ and polynomials of degree ranging from one to seven

Analysis

A digital computer program has been written to calculate the best (in a least-squares sense) values for the coefficients for each of the seven polynomials considered A print-out is made of the coefficients and the root-mean-square difference between the data points and the polynomials A similar routine is followed for the equation $p = k_\phi z^n$, except that the transformation $x = \log p$ and $y = \log z$ is made, which is equivalent to plotting the data on log-log paper and fitting a straight line to the data thus plotted It should be noted that this transformation compresses the data at high values, in exactly the same manner as a plot of the data on log-log paper This amounts to a bias, or weighting, of the low values of pressure and sinkage

Results

A total of 51 sets of soil data was considered In every case, a third-degree polynomial $z = a_1p + a_2p^2 + a_3p^3$ gave

Received December 10, 1963; revision received January 10, 1963

* Staff Engineer, Defense Operations Division

a smaller root-mean-square (rms) error than $p = k_\phi z^n$ In 46 cases of the 51, a second-degree polynomial $z = a_1p + a_2p^2$ gave a smaller root-mean-square (rms) error than $p = k_\phi z^n$ In 23 cases of the 51, a first-degree polynomial $z = a_1p$ gave a better fit than $p = k_\phi z^n$—a surprising result, since, by taking $n = 1$, the two equations are identical in form The difference arises from the bias in the logarithmic transformation

Figure 1 illustrates a comparison for one of the soils tested The graph shows the original x-y data plot together with the curves predicted by two of the equations Table 1 is a tabulation of the rms error involved with each approximation for the data of Fig 1

Table 1 Tabulation of rms error associated with each equation

Equation	rms error, in $[(z - \bar{z})^2/N]^{1/2}$
$p = k_\phi z^n$	0 2802
$z = a_1p$	0 0433
$z = a_1p + a_2p^2$	0 0383
$z = a_1p + a_2p^2 + a_3p^3$	0 0186
$z = a_1p + a_2p^2 + a_3p^3 + a_4p^4$	0 0081
$z = a_1p + a_2p^2 + a_3p^3 + a_4p^4 + a_5p^5$	0 0062
$z = a_1p + a_2p^2 + a_3p^3 + a_4p^4 + a_5p^5 + a_6p^6$	0 0043
$z = a_1p + a_2p^2 + a_3p^3 + a_4p^4 + a_5p^5 + a_6p^6 + a_7p^7$	0 0043

Conclusions

Any polynomial of degree two or higher gives a better fit to the majority of the data considered than the power function now in use In addition, it is anticipated that a polynomial would give a better fit to data from nonhomogeneous soils, such as soft soil over hardpan The only practical question lies in where the polynomial should be truncated This truncation is simply a tradeoff between accuracy and complexity Two or possibly three terms would seem to be the appropriate compromise

One distinct disadvantage of the equation $p = k_\phi z^n$ is that k_ϕ has dimensions (force)/(length)$^{n+2}$ That is, its dimensions depend upon the shape of the pressure sinkage curve Thus, different scaling laws are required for different soils The polynomial does not suffer this disadvantage

References

[1] Hackman, R J and Mason, A C , "Engineer special study of the surface of the moon," U S Geological Survey, Washington, D C (1961)

[2] Kopal, Z , *Physics and Astronomy of the Moon* (Academic Press, Inc , New York, 1962), p 154

[3] Hapke, B , "Photometric and other laboratory studies re lating to the lunar surface," Center for Radiophys and Space Res , Cornell Univ ; also Lunar Surface Materials Conf , Bos ton, Mass (May 1963)

[4] Lawrence, L , Jr and Lett, P W , "Characterization of lunar surfaces and concepts of manned lunar roving vehicles," Soc Automotive Engrs Paper 632L (January 1963)

[5] Bekker, M G , *Off the Road Locomotion, Research and Development in Terramechanics* (University of Michigan Press, Ann Arbor, Mich , 1960), p 33

[6] Halajian, J D , "Laboratory investigation of moon soils," IAS Paper 62-123, p 53 (June 1962)

[7] Rowe, R D and Selig, E T "Penetration studies of simulated lunar dust," Seventh Symposium on Ballistic Missile and Aerospace Technology, p 13

[8] Wismer, R D and Smith, M E , ' Vehicle systems, discussion, Proceedings of the 1st International Conference on the Mechanics of Soil, pp 762–769

[9] Buck, R C , *Advanced Calculus* (McGraw-Hill Book Co , Inc New York, 1956), p 39

Proton Fluxes along Low-Acceleration Trajectories through the Van Allen Belt

FRANK J HRACH*
NASA Lewis Research Center, Cleveland, Ohio

Introduction

THE radiation hazard of energetic protons trapped in the earth's magnetic field becomes an important consideration for manned, low-thrust, interplanetary vehicles because of the long period of time that such a vehicle must spend in the region near the earth The advantage of high-payload ratios attainable with low-thrust electrically propelled vehicles may be seriously offset by the necessity of increased weight to protect against Van Allen radiation

An estimate of the integrated dose received during a high-thrust lunar trajectory through the radiation belts behind various thicknesses of carbon shield has been made [1] Also, a method of computing total time-integrated proton flux for an arbitrary trajectory through the inner Van Allen belt has been formulated, and examples of integrated flux for circular orbits are offered in Ref 2 An approximate calculation of the upper and the lower limits of the integrated proton flux encountered during a low-thrust departure from an earth orbit was made[3] for a thrust-to-weight ratio of 10^{-4}

This paper presents an estimate of the time-integrated flux for trajectories having a constant acceleration in the range of 5×10^{-4} to 10^{-2} m/sec^2 and an inclination (angle between the vehicle orbit plane and the equatorial plane) in the range of 0° to 90°

Spatial Distribution of Flux

The distribution of the proton flux contours was assumed to be that given in Ref 3 and is reproduced in Fig 1 Flux contours of protons with energies greater than 40 Mev are plotted in the geomagnetic plane The maximum intensity is 4×10^4 protons/cm^2-sec This would be the actual distribution in space if the earth's magnetic field were a perfect earth-centered dipole field This, of course, is not the case The distribution is distorted to various degrees for different values of longitude The variations in latitude and altitude of the base point in the figure are given in Ref 3 It was assumed that every point in the distribution was rotated through the same angle and translated through the same distance as the base point for that particular longitude This assumption implies that the central region of the distorted field can be represented by the central portion of a dipole field

The approximation is better than the assumption that the earth's magnetic field can be represented by a displaced dipole field but is inferior to a description of the field based on a spherical harmonic analysis It is felt, however, that present-day uncertainties in the location of the upper-altitude contours in the dipole plane and in the temporal variations of the contour locations do not warrant the computer time required for low-thrust calculations based on an accurate field description

Vehicle Trajectory

The vehicle was assumed to have constant tangential acceleration throughout its passage through the belt The results are also applicable with little error to vehicles employing constant-thrust engines since the change in mass during this period is quite small for the specific impulses typical of electric propulsion

Received December 20, 1963

* Aerospace Engineer, Mission Analysis Branch Member AIAA